1.

$2a + 3c = 20 \quad \times 2 \quad : \quad 4a + 6c = 40 \quad (1)$

$4a + 4c = 34. \quad (2)$

$(1) - (2) : \quad 2c = 6$

$c = 3.$

$2a + 3(3) = 20$

$a = \frac{11}{2}$

$6a + 2c = 6\left(\frac{11}{2}\right) + 2(3) = \frac{66}{2} + 6 = 33 + 6$

$= \underline{£39}$

$\boxed{D}$

2.

- $2n - 5 = 17$

  $2n = 22$

  $n = 11$: 11^{th} TERM

- TERM-TO-TERM RULE : ADD 2

$\boxed{G}$

3. $Pr(1) = \frac{3}{8}$

$Pr(2) = \frac{1}{8}$

$Pr(3) = \frac{4}{8}$

NEED 2 AND 3

$Pr(2 \text{ THEN } 3) = \frac{1}{8} \times \frac{4}{8} = \frac{4}{64} = \frac{1}{16}$

$Pr(3 \text{ THEN } 2) = \frac{4}{8} \times \frac{1}{8} = \frac{4}{64} = \frac{1}{16}$

$$\frac{1}{8}$$

$\boxed{A}$

4. AREA OF SQUARE $= x^2$

AREA OF WHITE SEMICIRCLE $= \dfrac{\pi r^2}{2} = \dfrac{\pi\left(\frac{x}{2}\right)^2}{2}$

$= \dfrac{\pi x^2}{8}$

AREA OF WHITE SHAPE IN TOP RIGHT OF SQUARE $=$ AREA OF SQUARE $-$ AREA OF SECTOR

$= x^2 - \frac{1}{4}\pi(x)^2$

$= x^2 - \dfrac{\pi x^2}{4}.$

SHADED AREA

$= x^2 - \left[\dfrac{\pi x^2}{8} + x^2 - \dfrac{\pi x^2}{4}\right]$

$= \dfrac{\pi x^2}{8}$

$\boxed{A}$

5. GEOMETRIC SEQUENCE

$a, ar, ar^2, ar^3, \ldots$

$a = 5000$ START

$ar = 5000 \times 0.4$ END OF WEEK 1

$ar^2 = 5000 \times 0.4^2$

$ar^3 = 5000 \times 0.4^3 = \underline{320}$ cm^3

$\boxed{C}$

6. $m_1 m_2 = -1$: PERPENDICULAR

1 : GRADIENT $m = -\frac{1}{3}$

2 : $m = \frac{1}{3}$

3 : $m = 3$

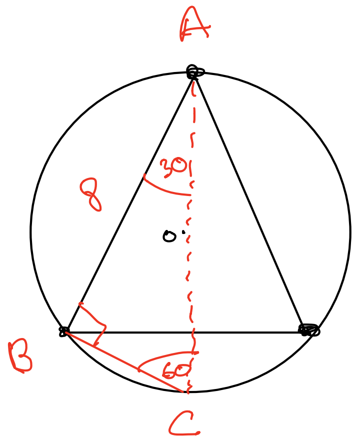
4 : $m = -\frac{2}{3}$

$-\frac{1}{3} \times 3 = -1$

$\therefore$ 1 AND 3 PERPENDICULAR

$\boxed{B}$

7.



- FORM TRIANGLE ABC
- ANGLE IN A SEMICIRCLE IS 90° SO $ABC = 90^\circ$ (CIRCLE THEOREM)
- $C\hat{A}B$ (INSIDE) $= \frac{1}{2} \times 60^\circ = 30^\circ$
- $\sin 60 = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}$

$$= \frac{8}{AC}$$

$$AC = \frac{8}{\sin 60}$$

$\therefore$ CIRCUMFERENCE $= \pi d$

$$= \frac{8\pi}{\sin 60^\circ}$$

$\boxed{B}$

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8. $2\left(\frac{x}{4}+3\right)^2 - \left(\frac{x}{4}+3\right) - 36 = 0.$

LET $y = \frac{x}{4} + 3$

$2y^2 - y - 36 = 0$

$(2y - 9)(y + 4) = 0$

$y = \frac{9}{2}, -4$

$y = \frac{x}{4} + 3$

$4y = x + 12$

$x = 4y - 12$

$x = 6, -28$

SUM $= -22$

$\boxed{F}$

9. $(2x+3)^2 - (x-3)^2$

$= 4x^2 + 12x + 9 - (x^2 - 6x + 9)$

$= 3x^2 + 18x$

$= 3(x^2 + 6x)$

$= 3\left[(x+3)^2 - 9\right]$

$= 3(x+3)^2 - 27$

$r = -27$ $\boxed{A}$

10. $\dfrac{a}{b/c} - \dfrac{a/b}{c}$

$= a\dfrac{c}{b} - \dfrac{a}{b}\dfrac{1}{c}$

$= \dfrac{ac}{b} - \dfrac{a}{bc}$

$= \dfrac{ac^2 - a}{bc} = \dfrac{a(c^2-1)}{bc}.$

$\boxed{E}$

11. MEAN = $\dfrac{\text{SUM OF SCORES}}{\text{NUMBER OF STUDENTS}}$

$= \dfrac{(10 \times 36) + (20 \times 48)}{10 + 20}$

$= 44$

RANGE IS AT LEAST MAXIMUM OF RANGE OF BOYS AND RANGE OF GIRLS

RANGE ≥ 21

$\boxed{F}$

12. LET P BE THE NUMBER OF PAIRS OF BOOTS

LET T BE OUTSIDE TEMPERATURE

- $P = \frac{k}{T^3}$, WHERE k IS A CONSTANT , $k > 0$.

- $250 = \frac{k}{8^3} = \frac{k}{512}$

$k = 250 \times 512 = 128{,}000$

- P = 700% MORE THAN 250

$= 250 + 7(250) = 2000$

$$2000 = \frac{128{,}000}{T^3}$$

$$T = \sqrt[3]{\frac{128000}{2000}} = \sqrt[3]{64} = 4$$

$\boxed{B}$

13. LET P BE PRE-SALE PRICE

S BE SALE PRICE

$$S = 0.75P \iff \frac{4}{3}S = P$$

CUSTOMER CALCULATES $1.25S$

$$|1.25S - P| = 15$$

$$\left|1.25S - \frac{4}{3}S\right| = 15$$

$$\left|\left(\frac{5}{4} - \frac{4}{3}\right)S\right| = 15$$

$$\left|\frac{15-16}{12}S\right| = 15$$

$$\frac{1}{12}S = 15$$

$$S = 15 \times 12 = 180$$

$$P = \frac{4}{3}S = \frac{4}{3}(180) = \underline{£240}$$

$\boxed{E}$

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14. R : B
18 : 5

B : 4
p : 3

R : 4
12 : 5
×1.5
18 : 7.5

R : B : 4
18 : 5 : 7.5

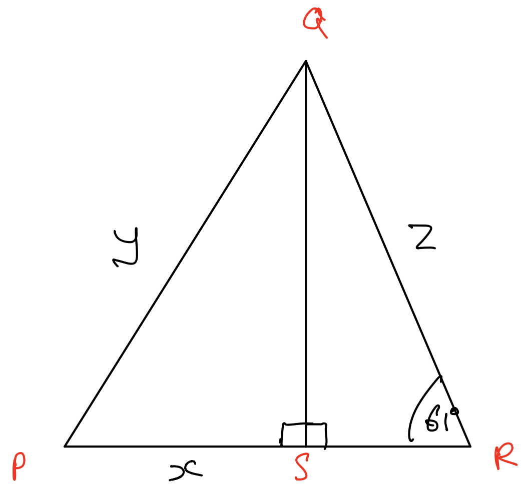
B : 4
2 : 3

$\times \frac{3}{7.5} = \times 0.4$

p = 2

A

15.



$$QS = \sqrt{y^2 - x^2}$$

$$\sin 61 = \frac{QS}{z} = \frac{\sqrt{y^2 - x^2}}{z}$$

$$z = \frac{\sqrt{y^2 - x^2}}{\sin 61°}$$

$\boxed{F}$

16.

P(AT LEAST ONE OF FOUR NUMBERS IS EVEN)

= 1 — P (ALL NUMBERS ARE ODD)

$= 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$ $\boxed{E}$

17. $2x^2 - px - 4 = 0$

QUADRATIC FORMULA:

$$x = \frac{p \pm \sqrt{p^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{p \pm \sqrt{p^2 + 32}}{4}.$$

$$x_2 - x_1 = \frac{p + \sqrt{p^2+32}}{4} - \left(\frac{p - \sqrt{p^2+32}}{4}\right)$$

$$= \frac{2\sqrt{p^2+32}}{4} = \frac{\sqrt{p^2+32}}{2} = 6.$$

$$\sqrt{p^2+32} = 12$$

$$p^2 + 32 = 144$$

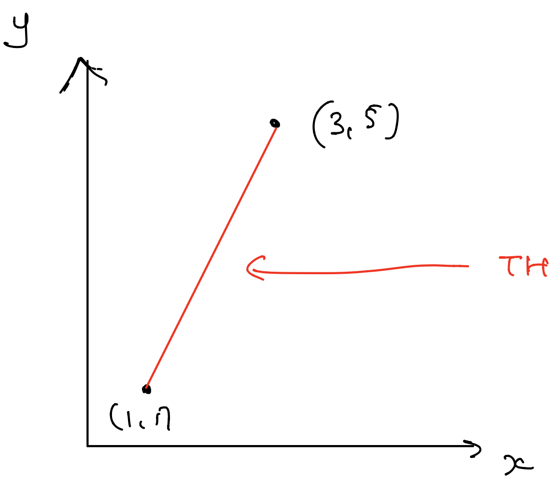
$$p^2 - 112 = 0$$

$$p = +\sqrt{112} = \sqrt{4 \times 28} = \sqrt{4 \times 4 \times 7}$$

$$p = 4\sqrt{7}$$

$\boxed{B}$

18.



THIS IS EITHER 1 : A SIDE OF A SQUARE

OR 2 : A DIAGONAL OF A SQUARE

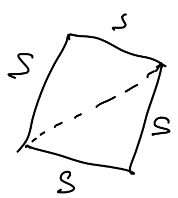
CASE 1 :

LENGTH BETWEEN POINTS $= \sqrt{(3-1)^2 + (5-1)^2}$

$= \sqrt{4 + 16} = \sqrt{20}$

PERIMETER $= 4\sqrt{20} = \underline{8\sqrt{5}}$

CASE 2 :



DIAGONAL LENGTH $= \sqrt{20}$

SIDE LENGTH s :

$s^2 + s^2 = (\sqrt{20})^2$

$2s^2 = 20$

$s^2 - 10 = 0$

$s = +\sqrt{10}$ $(s \geq 0)$

PERIMETER $= 4\sqrt{10}$

DIFFERENCE : $8\sqrt{5} - 4\sqrt{10} = 4\sqrt{5}\,(2 - \sqrt{2})$ $\boxed{D}$

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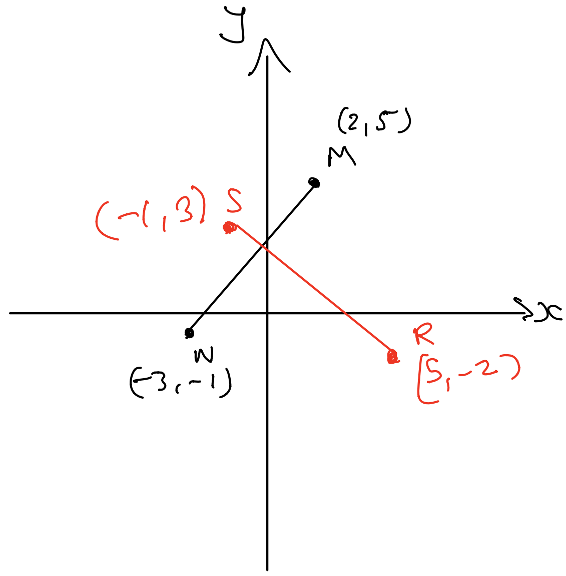
19.



STEP 1

STEP 2

TU HAS END POINTS $(-1+p,\ 3+q)$, $(5+p, -2+q)$

MIDPOINT $= (7, -2.5)$

$$= \left(\frac{-1+p+5+p}{2}, \frac{3+q-2+q}{2}\right)$$

$$= \left(\frac{4+2p}{2}, \frac{1+2q}{2}\right)$$

$$= \left(2+p, \frac{1}{2}+q\right)$$

$p = 5$, $\frac{1}{2}+q = -\frac{5}{2}$

$q = -3$

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ $\boxed{C}$

20. • RADIUS $= x$

• PERPENDICULAR HEIGHT $= \frac{5}{2}x$.

• VOLUME OF CONE $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$

VOLUME OF HEMISPHERE $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{y}{2}\right)^3 = \frac{1}{12}\pi y^3$

• $\frac{\text{VOLUME OF CONE}}{\text{VOLUME OF HEMISPHERE}} = \frac{\frac{5}{6}\pi x^3}{\frac{1}{12}\pi y^3} = \underline{\underline{\frac{10x^3}{y^3}}}$ $\boxed{b}$

21. $\int_4^9 f(x)\,dx = \left[F(x)\right]_4^9$

$g(x) = 2f(x) + 1$

$G(x) = \int (2f(x) + 1)\,dx$

$= 2F(x) + x + C.$

$\int_4^9 g(x)\,dx = \left[G(x)\right]_4^9 = \left[2F(x) + x\right]_4^9$

$= (2F(9) + 9) - (2F(4) + 4)$

$= 2(F(9) - F(4)) + 9 - 4$

$= 2(3) + 5 = \underline{11}$ $\boxed{F}$

22.

REMAINDER THEOREM

LET $f(x) = x^4 + ax^3 + bx^2 - 12x + 4$

$f(1) = f(2) = 0$

$f(1) = 1 + a + b - 12 + 4 = a + b - 7 = 0$ ①

$f(2) = 16 + 8a + 4b - 24 + 4$

$= 8a + 4b - 4 = 0$

$2a + b - 1 = 0$ ②

② − ①: $a + 6 = 0$

$\underline{a = -6}$

$a + b - 7 = 0 \iff b = 7 - a$

$= 7 - (-6)$

$\underline{b = 13}$

$\boxed{B}$

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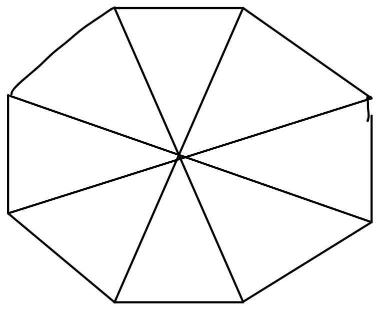
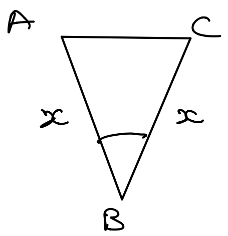
23.



OCTAGON CAN BE SPLIT INTO 8 IDENTICAL TRIANGLES

TAKE ONE TRIANGLE:



AREA $= \frac{32\sqrt{2}}{8} = 4\sqrt{2}$ cm^2

ANGLE ABC (INSIDE TRIANGLE)

$= \frac{360}{8} = 45°.$

AREA $= \frac{1}{2}x^2 \sin 45 = \frac{1}{2}x^2 \frac{\sqrt{2}}{2} = 4\sqrt{2}$

$\frac{x^2}{4} = 4$

$x^2 = 16$

$x = 4 \quad (x > 0)$

PT $= 2x = 8$

$\boxed{A}$

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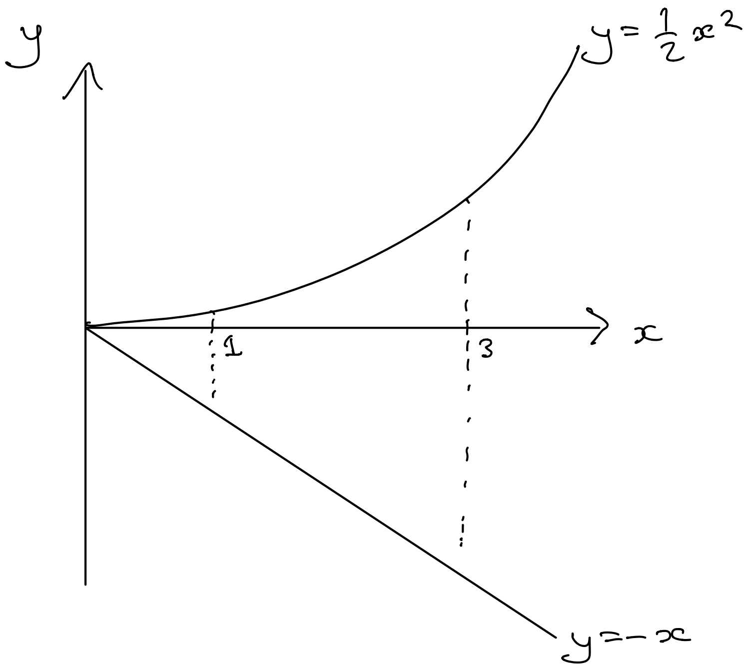
24.



$$\int_1^3 \left(\frac{1}{2}x^2 - (-x)\right) dx$$

$$= \int_1^3 \left(\frac{1}{2}x^2 + x\right) dx = \left[\frac{x^3}{6} + \frac{x^2}{2}\right]_1^3$$

$$= \frac{27}{6} + \frac{9}{2} - \left(\frac{1}{6} + \frac{1}{2}\right)$$

$$= \frac{26}{6} + 4 = \frac{50}{6} = \frac{25}{3}$$

$\boxed{E}$

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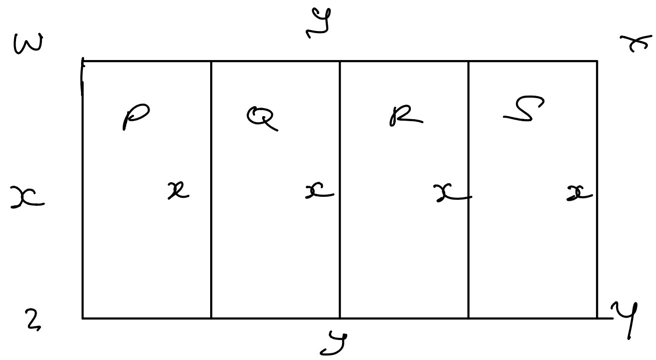
25.



LET $WZ = x$, $WX = y$

TOTAL LENGTH OF WALL $= 5x + 2y = 260$.

AREA OF P $= x\frac{y}{4}$

MAXIMISE $\frac{xy}{4}$ SUBJECT TO $5x + 2y = 260$

SUBSTITUTE: AREA $= \dfrac{x\left(\frac{260 - 5x}{2}\right)}{4}$

$= \dfrac{x(260 - 5x)}{8}$.

$\dfrac{d\,\text{AREA}}{dx} = \dfrac{1}{8}\left[260 - 10x\right] = 0$

$260 = 10x$

$x = WZ = \underline{26M}$ $\boxed{B}$

26.

STRAIGHT LINE $y = mx + c$

REFLECT IN LINE $y=x$ (INVERSE)

FIND INVERSE : $\frac{y-c}{m} = x$

NEW LINE $y = \frac{x-c}{m}$

GRADIENT $= \frac{1}{m}$

$\boxed{C}$

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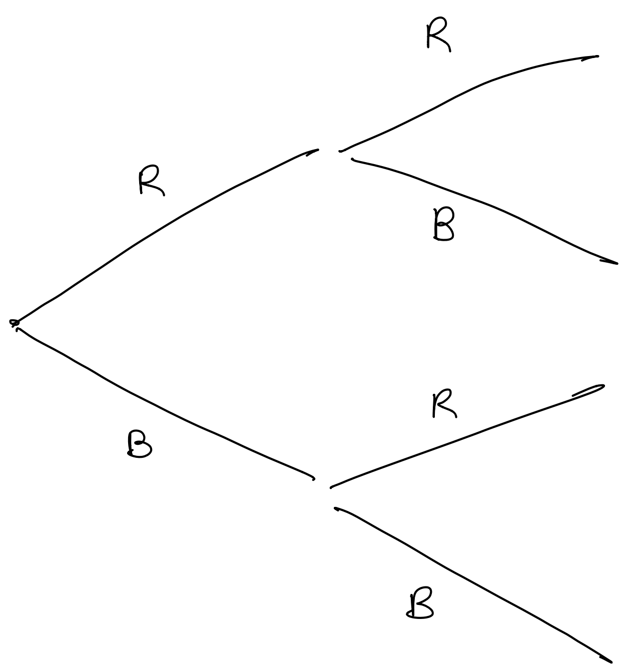
27. $R = \text{RED}$

$B = \text{BLUE}$

PROBABILITY :



$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$

$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$$P(1\,B \mid \text{AT LEAST } 1\,R) = \frac{P(1\,B \cap \text{AT LEAST } 1\,R)}{P(\text{AT LEAST } 1\,R)}$$

$$= \frac{\frac{2}{6} + \frac{2}{6}}{\frac{2}{6} + \frac{2}{6} + \frac{1}{6}} = \frac{\frac{4}{6}}{\frac{5}{6}} = \frac{4}{5}$$

$\boxed{C}$

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28. ARITHMETIC PROGRESSION:

$a, \quad a+d, \quad a+2d, \ldots$

· SUM OF FIRST n TERMS:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

· $S_{20} = 50 = \frac{20}{2}\left[2a + 19d\right]$

$50 = 10\left[2a + 19d\right] \qquad 5 = 2a + 19d \quad$ (A)

· $S_{40} - S_{20} = -50 \qquad S_{40} = S_{20} - 50 = 50 - 50 = 0.$

$S_{40} = 0 = \frac{40}{2}\left[2a + 39d\right]$

$2a + 39d = 0 \quad$ (B)

(A) − (B):

$5 = -20d$

$\underline{-\frac{1}{4} = d}, \qquad a = -\frac{39}{2}d$ FROM (B)

$= -\frac{39}{2}\left(-\frac{1}{4}\right) = \frac{39}{8}$

· $S_{100} = \frac{100}{2}\left(2a + 99d\right)$

$= 50\left(2\left(\frac{39}{8}\right) + 99\left(-\frac{1}{4}\right)\right)$

$= \underline{-750}$

$\boxed{A}$

29. $x_{n+1} = -\frac{12}{x_n}$. $\Longleftrightarrow$ $x_n = -\frac{12}{x_{n+1}}$

$x_{50} = 6$.

$x_{49} = -\frac{12}{x_{50}} = -2$.

$x_{48} = -\frac{12}{x_{49}} = -\frac{12}{-2} = 6$

n even, $x_n = 6$
n odd, $x_n = -2$

SUM OF FIRST 15 TERMS

$= 8(-2) + 7(6) = -16 + 42 = \underline{26}$

$\boxed{E}$

30.

- SUBSTITUTE $(x, y) = (2, 4)$ INTO $y = mx + c$

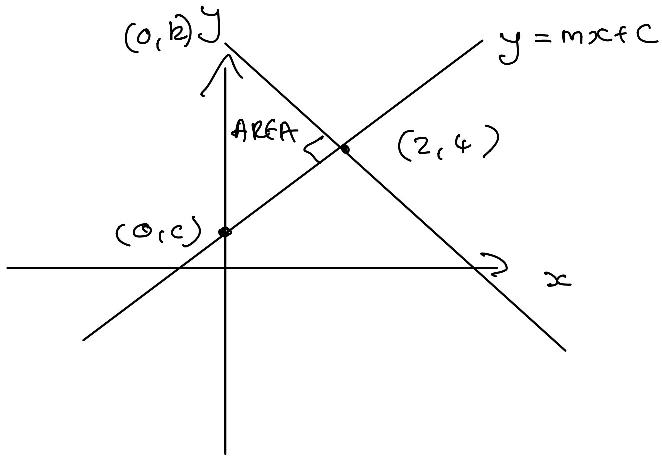
$$4 = 2m + c$$

- PERPENDICULAR LINE : GRADIENT $-\frac{1}{m}$

$$y = -\frac{1}{m}x + k$$

$$4 = -\frac{1}{m}(2) + k$$

$$4 + \frac{2}{m} = k.$$



$$\text{AREA} = \frac{1}{2}bh = \frac{1}{2}(k - c)(2)$$

$$= \frac{1}{2}\left(4 + \frac{2}{m} - (4 - 2m)\right)(2)$$

$$= 2m + \frac{2}{m} = 5$$

×m

$$2m^2 - 5m + 2 = 0$$

$$(2m - 1)(m - 2) = 0$$

$$m = \frac{1}{2}, 2$$

$\boxed{D}$

31. LET NUMBERS BE x_i, $i = 1, \ldots, n$.

MEAN $= \frac{1}{n} \sum_{i=1}^{n} x_i = m$. (1)

UNIQUE MODE CHANGES $\rightarrow$ 2 NUMBERS REMOVED ARE BOTH EQUAL TO UNIQUE MODE d.

MEAN OF REMAINING NUMBERS

$$= \frac{1}{n-2} \Big(\underbrace{\sum_{i=1}^{n} x_i}_{= mn. \text{ FROM } (1)} - 2d \Big) = m + 2.$$

$$mn - 2d = (m+2)(n-2)$$
$$= mn + 2n - 2m - 4.$$
$$-d = n - m - 2$$
$$\underline{d = m + 2 - n}$$

$\boxed{E}$

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32. P : $a, ar, ar^2, \ldots$

Q : $b, bs, bs^2, \ldots$

$ar^2 = 4 = bs^2$

$ar^4 = 2 = bs^4$

$r^2 = s^2 = \frac{2}{4} = \frac{1}{2}$

$r = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ $\quad s = -\frac{1}{\sqrt{2}}$ (WE WILL ASSUME $\sqrt{2}$ IS THE POSITIVE ROOT)

$a = \frac{4}{r^2} = \frac{4}{\left(\frac{1}{2}\right)} = 8 = b$

$$S_\infty^P - S_\infty^Q = \frac{a}{1-r} - \frac{b}{1-s} = \frac{8}{1-\frac{1}{\sqrt{2}}} - \frac{8}{1+\frac{1}{\sqrt{2}}}$$

$$= \frac{8}{\frac{\sqrt{2}-1}{\sqrt{2}}} - \frac{8}{\frac{\sqrt{2}+1}{\sqrt{2}}}$$

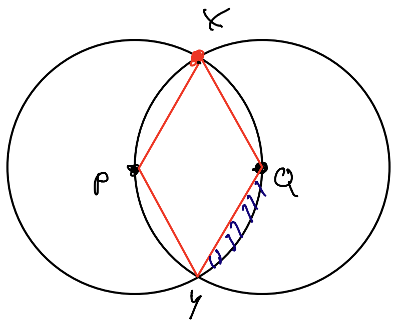
$$= \frac{8\sqrt{2}}{\sqrt{2}-1} - \frac{8\sqrt{2}}{\sqrt{2}+1}$$

$$= \frac{8\sqrt{2}(\sqrt{2}+1)}{2-1} - \frac{8\sqrt{2}(\sqrt{2}-1)}{2-1}$$

$$= 8\sqrt{2} + 8\sqrt{2} = 16\sqrt{2}$$

$\boxed{E}$

33.



SPLIT INTO TRIANGLES + SEGMENTS OF CIRCLE

PQ = 1 cm

XQ = PX = 1 cm (RADII)

SO PXQ EQUILATERAL TRIANGLE

AREA OF PXQ TRIANGLE $= \frac{1}{2}ab\sin C = \frac{1}{2}(1)(1)\sin 60$

$= \frac{1}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

AREA OF RED QUADRILATERAL PXQY

$= 2\ PXQ = \frac{\sqrt{3}}{2}$

AREA OF SHADED SEGMENT

$$= \underbrace{\pi r^2 \frac{60}{360}}_{\text{AREA OF SECTOR}} - \underbrace{\frac{\sqrt{3}}{4}}_{\text{AREA OF PQY TRIANGLE}} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}.$$

TOTAL AREA = AREA OF RED QUADRILATERAL + 4 (AREA OF SHADED SEGMENT)

$$= \frac{\sqrt{3}}{2} + 4\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{4\pi}{6} - \sqrt{3} = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \quad \boxed{D}$$

34. FIND STATIONARY POINTS BY DIFFERENTIATING & SET TO ZERO:

$$\frac{dy}{dx} = 3x^2 + 6\sqrt{5}\,px + 3p = 0.$$

$$x^2 + 2\sqrt{5}\,px + p = 0$$

QUADRATIC FORMULA

$$x = \frac{-2\sqrt{5}\,p \pm \sqrt{(2\sqrt{5}\,p)^2 - 4(1)(p)}}{2(1)}$$

$$x = \frac{-2\sqrt{5}\,p \pm \sqrt{20p^2 - 4p}}{2}$$

$$x = -\sqrt{5}\,p \pm \sqrt{5p^2 - p}$$

NEED TWO SOLUTIONS, SO NEED $\sqrt{5p^2 - p} > 0$

$$5p^2 - p > 0$$

$$p(5p-1) > 0$$

$p = 0,\ \frac{1}{5}$ CRITICAL VALUES

$$p < 0,\quad p > \frac{1}{5}$$

$\boxed{A}$

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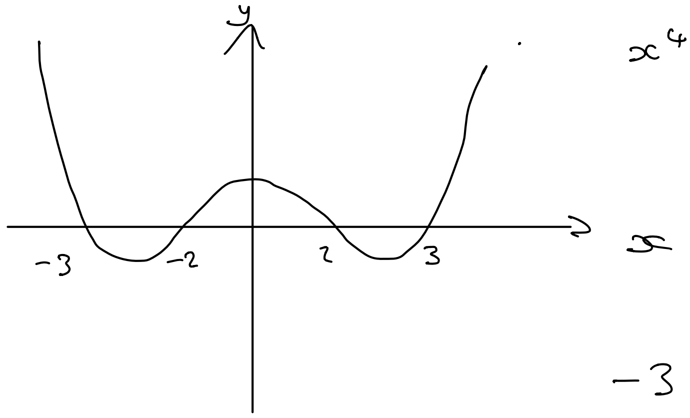
35.

$$x^4 - 13x^2 + 36 < 0$$

$$(x^2 - 9)(x^2 - 4) < 0$$

CRITICAL VALUES : $x = \pm 3, \pm 2.$



x^4 GRAPH HAS THIS SHAPE

$-3 < x < -2, \quad 2 < x < 3$

$\boxed{G}$

36. $\sin^2 x = 1 - \cos^2 x$

SUBSTITUTE :

$$14\cos^3 x + 10(1-\cos^2 x)\cos x = 13\cos x.$$

$$4\cos^3 x - 3\cos x = 0$$

$$\cos x(4\cos^2 x - 3) = 0$$

$\cos x = 0$ — 4 SOLUTIONS (2 SOLUTIONS EVERY 2π)

$4\cos^2 x - 3 = 0$

$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$ — 8 SOLUTIONS

(2 SOLUTIONS EVERY 2π FOR $\cos x = +\frac{\sqrt{3}}{2}$, 2 SOLUTIONS EVERY 2π FOR $\cos x = -\frac{\sqrt{3}}{2}$)

$4 + 8 = 12$ SOLUTIONS

$\boxed{E}$

37.

$$2^x = 3^{2-x}$$

TAKE LOGS :

$$\log 2^x = \log 3^{2-x}$$ [LOGS ARE IN BASE 10]

$$x \log 2 = (2-x)\log 3$$

$$x(\log 2 + \log 3) = 2\log 3$$

$$x = \frac{2\log 3}{\log 2 + \log 3} = \frac{\log(3^2)}{\log(2\times 3)} = \frac{\log 9}{\log 6}$$

$\boxed{F}$

38.

LET $\log_{10} x = \log x$.

$$(2\log x)^2 + \log x - 3 = 0$$

$$4(\log x)^2 + \log x - 3 = 0$$

$$(4\log x - 3)(\log x + 1) = 0$$

$\log x = \frac{3}{4}$ $\log x = -1$

$x = 10^{\frac{3}{4}}$ $x = 10^{-1}$

PRODUCT OF ROOTS: $10^{\frac{3}{4}} \times 10^{-1} = \underline{\underline{10^{-\frac{1}{4}}}}$ $\boxed{D}$

39.

DISTANCE BETWEEN $(0, \frac{9}{2})$ and $(x, \underbrace{x^2}_{y=x^2})$

$$= \sqrt{(x-0)^2 + (x^2 - \tfrac{9}{2})^2}$$

$$= \sqrt{x^2 + (x^2 - \tfrac{9}{2})^2}$$

MINIMISE DISTANCE = MINIMISE $x^2 + (x^2 - \frac{9}{2})^2$

$$x^2 + (x^2 - \tfrac{9}{2})^2 = x^2 + x^4 - 9x^2 + \tfrac{81}{4}$$

DIFFERENTIATE WRT x:

$$2x + 4x^3 - 18x = 0.$$

$$4x^3 - 16x = 0$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x = 0, 2, -2.$$

$$y = 0, 4, 4.$$

$\underline{y = 4}$ $\boxed{D}$

NB DIFFERENTIATE AGAIN:

$12x^2 - 16$.

AT $x = 0$, SECOND DERIVATIVE < 0 $\therefore$ LOCAL MAXIMUM DISTANCE

AT $x = \pm 2$, SECOND DERIVATIVE > 0 $\therefore$ (LOCAL) MINIMUM DISTANCE.

$\therefore$ IGNORE $x = 0, y = 0$.

40. $\frac{dy}{dx} = -4 + 6x^{\frac{1}{2}} - 2x$

MAXIMISE $\frac{dy}{dx}$: DIFFERENTIATE AGAIN :

$$\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}} - 2 = 0.$$

$$\frac{1}{\sqrt{x}} = \frac{2}{3}$$

$$\frac{3}{2} = \sqrt{x}$$

$$\underline{\frac{9}{4} = x}$$

AT $x = \frac{9}{4}$, $\frac{dy}{dx} = -4 + 6\left(\frac{9}{4}\right)^{\frac{1}{2}} - 2\left(\frac{9}{4}\right)$

$$= -4 + 6\left(\frac{3}{2}\right) - \frac{9}{2}$$

$$= -4 + 9 - \frac{9}{2}$$

$$= -4 + \frac{9}{2} = \underline{\frac{1}{2}}$$

$\boxed{C}$